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Persistent Time Series: An Application to Music Classification

ABSTRACT

Background

A time-varying system can be interpreted as a series of relevant geometric and topological events. Persistent homology has mainly been applied to the study of static point clouds and shapes, by providing a description of both the geometry and topology of the analysed space. See Frosini and Landi (2001), and Edelsbrunner and Harer (2009). When applied to the study of static spaces, one of the reasons that makes persistent homology so effective is that it provides a representation in which the features of the space result arranged by relevance. Thus, the analysis can be tuned on a specific application need, by balancing the computational cost and the level of details to retrieve. In particular, such fingerprint can be visualised as a *persistent diagram.* The feature of a space are represented in a persistence diagram as points and lines in the plane, ranked according to their distance from the diagonal. The two main ingredients in persistent homology are a filtration of the space one wants to analyse and a pairing of (homological) critical values of the function. A filtration is a sequence of nested spaces generally induced by the collection of sublevel sets of a continuous function defined on analysed space. The pairing is induced by the birth and death of homological classes along the filtration.

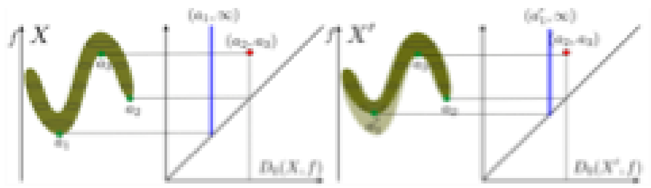

Fig. 1, Left: The height function on the topological space *X*, and its persistence diagram $D_0(X, f)$. Right: A variation of the geometry of the shape corresponds to an update of the persistence diagram.

Example 1. Consider the manifold in Figure 1 (left) and the height function *f* defined with respect to the axis depicted in the figure. The filtration is given by the sublevel sets of *f*, and the critical points are the maxima and minima of *f*. The *0-persistence diagram* $D_0(X, f)$ describes how the connected components of the shape are born and die along the filtration. In the figure a first connected component is born at height a_1 and will never die along the filtration: the resulting pairing is (a_1, ∞). A second connected component is born in a_2 and it will die when reaching the sublevel set defined by a_3, generating the pairing (a_2,a_3). In particular, it is possible to define a *k*-persistence diagram for every integer *k*. See Edelsbrunner and Harer (2009) for more details.

Persistent homology has been generalised to time-varying systems, either by considering continuous representations (Cohen-Steiner 2006), or introducing statistics (Munch 2013; Turner *et al*. 2014).

A time series is a collection of values obtained through subsequent, evenly sampled, observations in time. Time-series data mining (Esling and Agon 2012) is an attempt to organise data, in furtherance of visualising their *contour*, avoiding negligible details and creating a consistent, interpretable representation. Due to their generality and flexibility, time series are extensively used in applications, e.g. classification, segmentation (Keogh *et al*. 2004), and supported by a strong theoretical framework (Keogh and Kasetty 2003).

We suggest to combine the scalability (among other properties) of persistent homology, and the notion of time series. We define a *persistence time series* as the collection of persistent diagrams obtained by defining a filtration function on a time-varying manifold. In particular we refer to observations of a persistence time series as persistence snapshots.

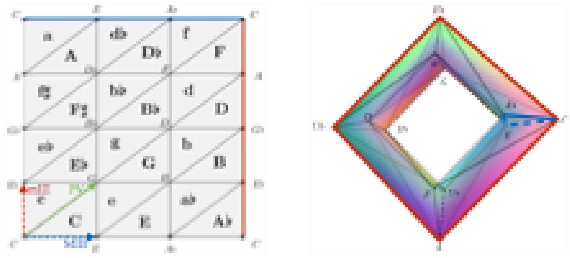

Fig. 2. On the left, a finite subcomplex of the planar *Tonnetz T*. On the right the *Tonnetz* torus.

Example 2. In Figure 2 (right) the geometrical variation of the absolute minimum of *X* determines an update of the associated persistent diagram.

This time-dependent representation allows to distinguish between relevant and noisy states in time by comparing diagrams associated with different observations. Persistence diagrams are indeed points of a metric space equipped with the bottleneck distance (Edelsbrunner and Harer 2009). In addition, the computation of the dissimilarity between time series (Liao 2005) allows to compare several time-varying systems; i.e. to find the timespans, if they exists, where two time-varying spaces can be considered comparable, in agreement with their time-dependent, geometric and topological characterisation. Music style analysis is a natural framework for this kind of formalism.

Aims and Repertoire Studied

We present a method to compare time-varying systems by taking advantage of their geometric and topological fingerprints expressed as persistence time series. If two spaces and their associated filtering functions are comparable, Dynamic Time Warping (DTW) is used to define an optimal warping

path between two *k*th persistence time series. An optimal warping path gives an encompassing view on the relative geometric behaviour of the analysed systems, by highlighting eventual irregular patterns in their geometric evolution. As an application we perform automatic stylistic clustering of three collections of classical, jazz and pop music.

Methods

We generalise to dynamical scenarios the model presented in Bergomi *et al*. (2016), in which persistent homology is used to characterise the stylistic content of musical compositions, represented as static 3-dimensional shapes. We take advantage of the topological representation of the *Tonnetz* as a simplicial complex (Bigo *et al*. 2013). We recall that *Tonnetz* can be seen on the one hand as a infinite planar simplicial complex we will denote as T, whose triangles represent minor and major triads. On the other hand as a torus, denoted as T in the reminder. The information concerning the harmonic relationships and the *temporal* hierarchy (durations) of notes in a musical phrase can be expressed by displacing the vertices of the *Tonnetz*. The vertices of T are labelled with pitch classes. Given a composition, we deform the planar *Tonnetz* T by displacing the vertices in height of a distance equal to the sum of the durations of the pitch class labelling the vertices. (See Figure 3.)

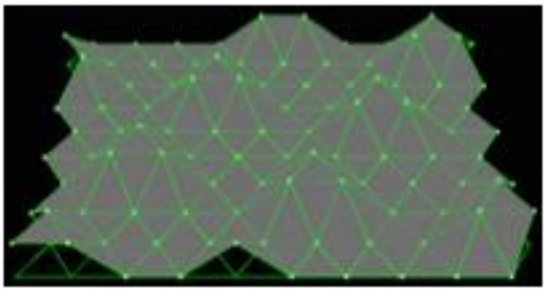

Fig. 3. The *Tonnetz* deformed with a major triad that appears as a 2-simplex (triangle) corresponding to a maximum of the height function.

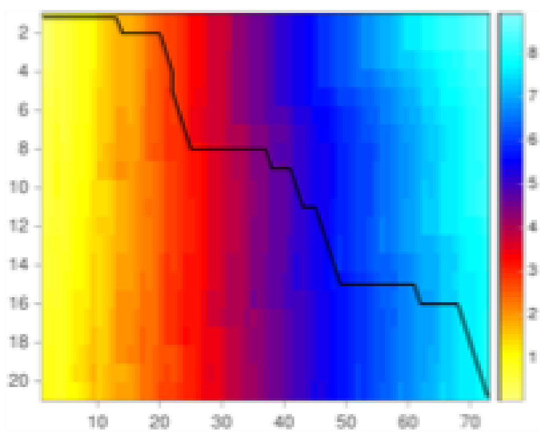

Fig. 4. Optimal warping path between two versions of *Caravan*. The positions of the gaps correspond to the solo parts of the longer version (frames 25–50 and 51–65 respectively).

The ordering induced by this displacement allows to define a filtration of the simplicial complex induced by the height function on the torus T. A 3-dimensional interactive animation showing how the *Tonnetz* is deformed by a musical phrase in time is available at <http:namilab.com/tonnetz/examples/deformed_tonnetz_int_sound_pers.html>.

Music is often organised in bars: modulations occur each four or eight bars in a jazz context, as well as the melodic line of the voice is arranged in a question and answer paradigm consisting of cycles of 2 or 4 bars in pop music. The idea is to take into account this natural segmentation to create a windowing of the composition.

By analysing the evolution in time of the persistence diagrams it is possible to detect relevant phenomena encoded in the progressive geometric update of the deformed *Tonnetz*. In Figure 5, a sequence of six 0th persistence diagrams computed considering an 8-bar windowing of Schoenberg's *Klavierstück I* is depicted. First, consider diagram of the first row of the figure: a line reveals the connected nature of T and represents the absolute minimum of the height function. This minimum corresponds to the subcomplex of the *Tonnetz* that is less used in the composition. The point highlights the presence of a second minimum of the height function associated to a subcomplex of T which is disconnected from the first one. In musical terms, the presence of these two connected components grasps the atonal nature of the piece: Disconnected subcomplexes of the *Tonnetz* are labelled with dissonant pitch-class sets (Cohn 1997). The lifespan of the point measures the relevance of this stylistic feature. The remainder of the observations describes the changes in terms of death and birth-levels of these connected components. Moreover, the increasing growth of the birth-levels of the points of the diagram represents the homogeneous gain of *height* of the entire simplicial complex in time. This fact means that the entire chromatic scale is uniformly used in the composition, both in terms of pitches and duration of the notes.

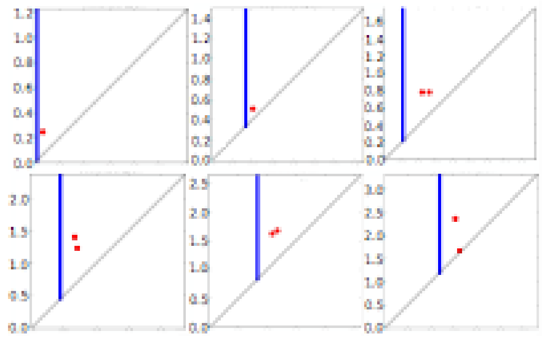

Fig. 5. The six first observation of the 0-persistence time series. *Klavierstück I* – Schoenberg. Persistence snapshots are taken each 8 bars.

In order to compare two compositions we measure the optimal alignment between the associated persistence time series, by using DTW. Dealing with persistence diagram, a reasonable choice as cost function is the bottleneck distance. In musical terms, an optimal warping path returns the comparable regions of the two compositions, represented by similar (near with respect to the bottleneck distance) persistence diagrams. In other words, time regions of the compositions that share a similar use of the entire set of pitch classes (point at infinity), or dissonant intervals both in terms of relevance (distance from the diagonal of the proper points), and in a balanced or unbalanced way (relative distance and multiplicity of the points), are aligned in an optimal warping path. We use DTW to compute the dissimilarity between 0th persistence time series associated with three datasets composed by classical, pop and jazz compositions, respectively.

Here we briefly discuss the analysis of two versions of *Caravan*. The presence of rich solos in a long version of the standard distinguishes it neatly by the second, shorter version in which only the theme is presented with small variations. Note how an optimal warping path between these two pieces depicted in Figure 4 tries to align them on the themes, skipping the solo parts. Hence, the evolution in time of the persistence diagrams grasps the difference between an organised thematic flow, and a freer improvisational context.

Keywords

Tonnetz, Style Recognition, Topology, Persistent Homology, Time Series, Dynamic Time Warping.

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