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The *Supplementum* in Renaissance Motets: Style and Structure

ABSTRACT

One common task in computational music analysis is the use of key-finding algorithms when examining common practice period music. The notion of key does not exist in earlier eras. Musicians in the Renaissance categorized music according to modes. A mode is a type of scale with a set of characteristic melodic behaviors. Applying key finding algorithms to modal literature only returns the following four keys: C major, A minor, F major, or D minor. Would it not be great to build a mode finding algorithm? To build a mode-finding algorithm, we need to evaluate methods of determining modal characteristics. One method used to determine mode is the 'final pitch'. But what is the final pitch of a polyphonic work? Is it the tenor? Is it the root within the final cadence? We need to look at the final cadence of a composition in order to answer these questions. However, sometimes the final cadence occurs before the end of a composition, and post-cadential material is added afterwards, a practice called a *supplementum*. When a piece has a *supplementum*, the sonority at the final cadence is not necessarily the pitch of the final sonority. We will discuss what computational tools were used for the ensuing corpus study. Then, we will look at how the *supplementum* was defined by historical and contemporary music theorists, how it is used in motets by different Renaissance composers, and how its use is different from descriptions of music theorists. Finally, we will postulate what we will be able to do with the information.

1. THE CORPUS

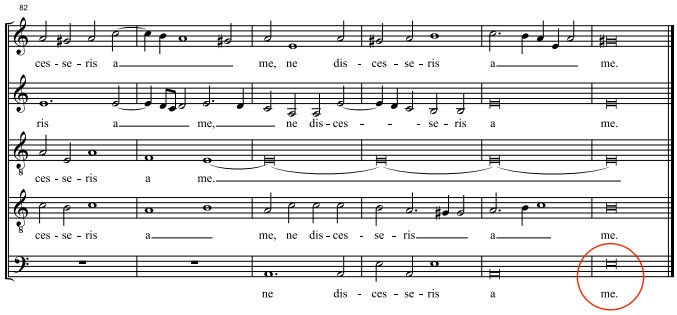

Fig. 1. Lassus, 'In me transierunt'.

To assemble the corpus, we collected 3,906 motets from curated online sources, assembled by research groups at universities, containing minimal amounts of mistakes, and non-curated online sources, assembled by everybody else, usually consisting of numerous mistakes, for example four sharps in a Renaissance motet, etc. All of the motets found online have to be encoded in a way that a computer can read them, meaning the music needs to be represented in one of the common symbolic music file formats. What is a symbolic music file format?

Figure 1 shows a graphically encoded file — readable by a human being — of 'In me transierunt' by Lassus. The file can be generated with any music Notation software such as Sibelius, Finale, MuseScore, LilyPond, etc. Figure 2 shows the musical content of the red circle in Figure 1, represented in a symbolically encoded music file, in most cases consisting of semantically organized text. A symbolic music file can be generated with one of the previously mentioned notation programs, or be written out with a text editor. There are many different types of symbolic music file formats with differing strengths and weaknesses. Figure 2 is a MusicXML representation of the red circle in Figure 1 'In me transierunt'.

The curated sources included in the following study are: 1) the ELVIS Database <http://database.elvisproject.ca/>, (accessed 25/06/2023) (ELVIS standing for Electronic Locator of Vertical Interval Successions), which is part of the Single Interface for Music Score Searching and Analysis (SIMSSA <https://simssa.ca/>, accessed 25/06/2023) project at McGill University; 2) the Josquin Research Project <http://josquin.stanford.edu/> (accessed 25/06/2023), assembled by Jesse Rodin in collaboration with Craig Sapp at the Center for Computer Assisted Research in the Humanities of Stanford University (CCARH <http://www.ccarh.org/>, accessed 25/06/2023); and 3) the Kern Scores web site <http://kern.ccarh.org/> (accessed 25/06/2023), also maintained by CCARH.

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Fig. 2. Symbolic Music Representation.

The non-curated sources included in the following study are: 1) the Choral Wiki <http://www3.cpdl.org/> (accessed 25/06/2023), assembled by choirs, singing enthusiasts, and other Liebhaber; 2) the Mutopia Project <http://www.mutopiaproject.org/> (accessed 25/06/2023), similar to the Choral Wiki, but including not just vocal music; 3) the MuseScore site <https://musescore.com/> (accessed 25/06/2023), a repository of music encoded in MuseScore's file format (MuseScore is free, open source notation software <https://musescore.org/> (accessed 25/06/2023); and 4) the newly established OpenScore site <https://www.openscore.cc/> (accessed 25/06/2023), aiming to to digitize all public domain sheet music.

Rather than going through the tedious process of downloading individual symbolically encoded music files from the aforementioned sites, a web scraper — software to automate the downloading process — developed by the SIMSSA project was utilized to expedite the task <https://github.com/ELVIS-Project/MediaWiki-Interface/> (accessed 25/06/2023). In addition, we limited the corpus to a date range from the 1470s to 1600s — i.e. the Renaissance period. The corpus includes the works of 492 composers.

2. TOOLS

The following computational tools were used to process all of the symbolically encoded music data. We used the Python programming language, which is a popular scripting and programming language amongst data scientists. We used Music21 <http://web.mit.edu/music21/> (accessed 25/06/2023), a Python-based toolkit for computational music analysis developed by Michael Cuthbert at MIT. We used pandas <https://pandas.pydata.org/> (accessed 25/06/2023), a Python-library, originally developed for the financial sector, providing high-performance, easy-to-use data structures and data analysis. We used the VIS-Framework <https://github. com/ELVIS-Project/vis-framework/> (accessed 25/06/2023), a library that combines music21 and pandas, developed for the analysis of contrapuntal music. The framework can be used to study vertical and horizontal interval successions in Renaissance and other music. We used the tools with a computer that had Python installed in conjunction with a text editor, with which analysis scripts were written.

3. *SUPPLEMENTUM* DEFINITION

Joachim Burmeister describes the *supplementum* in his *Musica poetica* (Burmeister 1606 [2007]). His conceptualization is picked up by other music theorists, and is identified by different terms such as the *manubrium* in Johannes Nucius's *Musices poeticae* (Nucius 1613), which occurs in 'virtually all motets', and the *paragoge* in Joachim Thuringus's *Opusculum bipartitium* (Thurignus 1624) which is nowadays employed in 'all compositions' (Bartel 1997). Thuringus adopted the term from Johannes Susenbrotus' *Epitome troporum* (Susenbrotus 1551), in which Susenbrotus lists the *paragoge* as one of the 132 rhetorical tropes and figures, describing it as an addition of a letter or syllable to an end of a word. Susenbrotus attributed the term and its definition to Antonio Mancinelli's *Carmen de Figuris*, chapter 20 (Mancinelli 1493). A century later Johann Gottfried Walther adopts Thuringus' *paragoge* term in his *Musicalisches Lexicon* (Walther 1732), and implies that the model was perhaps an improvisatory practice.

Returning to Burmeister, the *supplementum* is a passage two or more measures long, expanding on a 'primary' or 'secondary' pitch after the final cadence to emphasize its finality (Burmeister and Rivera 1993). The primary pitch is the goal of a cadence, i.e. the *finalis*, whereas the secondary pitch will most of the time be the *repercussa*. As Burmeister clarifies that the *supplementum* is an 'elaboration of a final pitch in a stationary voice', and that added pitches in other voices should create 'consonances with it' (Burmeister and Rivera 1993). Students, Burmeister prescribes, should study examples by master composers, and he provides a complete analysis of Lassus's 'In me transierunt' (Burmeister and Rivera 1993) to lead the way. The motet by Lassus includes a *supplementum*, and Claude Palisca provides an annotated version of Burmeister's analysis of the composition (Palisca 1972).

Contemporary scholars complement Burmeister's definition, and show *supplementa* in analyses of works by other composers. Julie E. Cumming labels a *supplementum* at the end of Isaac's 'Inviolata integra et casta es Maria' (Cumming 2011). Martin Just remarks on a supplementum in the 'Circumdederunt me (III/2)' of Jean Richafort's Requiem (Just 1990). Bernhard Meier points to a *supplementum* in Claudio Merulo's 'Toccata I, 1: *Primo tono*' (Meier 1977). Patrick McCreless identifies a *supplementum* in verse 13 from Lassus's setting of Psalm 143 in 'Seven Penitential Psalms' (McCreless 2008). Peter N. Schubert shows *supplementa* at the end of Palestrina's motets 'Dies sanctificatus' (Schubert 1993), and 'Benedicta sit' (Schubert 2007), and at the end of Lassus's Kyrie in 'Missa Je suis déshéritée' (Schubert and Lessoil-Daelman 2013).

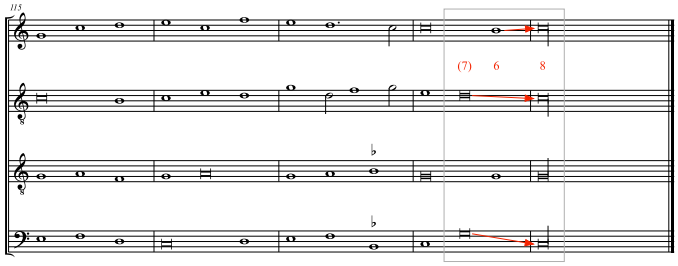

Fig. 3. Ockeghem, 'Alma redemptoris mater'.

Let us observe two musical examples, one without a *supplementum*, and one with a *supplementum*. We begin with Ockeghem's 'Alma redemptoris mater', a motet ending without a *supplementum*, or simply a final cadence (Figure 3). The voices from the top to the bottom are Cantus, Altus, Tenor, and Bassus. A sixths-to-octave (6–8) vertical interval succession, preceded by a 7–6 suspension, leads to a C between the Cantus and Altus parts. Melodically, the C is approached by an ascending half step from B in the Cantus part, whereas a descend by step from D to C occurs in the Altus part. The figure is supported with a descending 5th motion from G to C in the Bassus part. The 6–8 motion can also occur as its inversion in form of the 3–1 motion, as is the case in Figure 4.

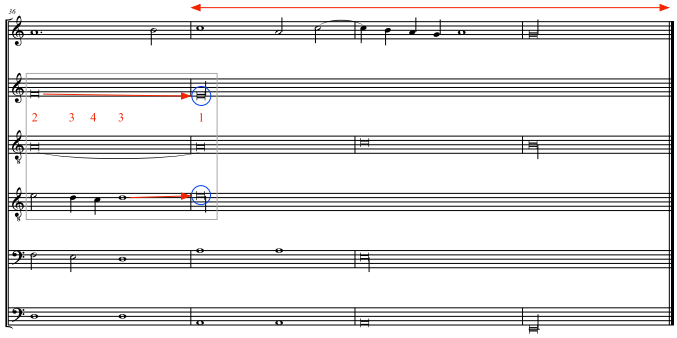

Fig. 4. Jean Mouton, 'Salve nos, Domine'.

Let us now examine a motet featuring a *supplementum*, and point to Burmeisters definitions. The *supplementum* appears in Moutons 'Salve nos, Domine' motet (Figure 4). The voices from top to bottom are: Superius, Altus, Tenor1, Tenor2, Bassus, and Bassus2. A final cadence occurs when a vertical motion from 3–1 is placed between the Altus, and Tenor2 parts that is supported by a half-step descending motion from F to E in the Altus part, and an ascending whole-step motion from a D

to an E in the Tenor2 part. We have an elaboration of the primary pitch E, which becomes the stationary pitch. We will revisit the example later in a slightly different context. The *supplementum* is 2+ measures long (2 breves), and the *supplementum* brings a sense of finality to the motet.

4. OVERALL OBSERVATIONS

Now that we know what a *supplementum* can look — and sound — like, let us examine statements of historical music theorists, by bringing in the data collected from the corpus. As you may recall Johannes Nucius asserted that virtually all motets have a *supplementum*.

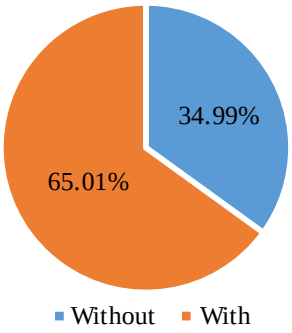

Fig. 5. Pieces with/without *Supplementa*.

The data backs Nucius' claim up (Figure 5), if by 'virtually all' we mean about 65 % (orange). However, what is it 65 % of? It turns out that Burmeister's assertion that a *supplementum* does occur only at the final cadence, and that it always signifies the finality of a composition is not quite accurate. *Supplementa* can occur at the end of any part of a multipart motet.

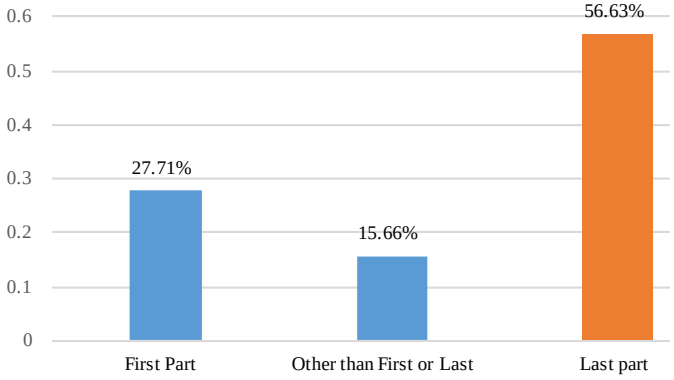

Fig. 6. Parts with *Supplementa*.

Therefore, our corpus actually expands by about 50 % — or to ca. 5,800 pieces —, since many of the motets have multiple parts. Out of the corpus with *supplementa* (Figure 6), 27,71 % occur in a *Prima pars*, and another 15,66 % occur in a part other than the first and last part. So, about 44 % of the *supplementa* do not occur at the end of a composition set. But all *supplementa* are 2+ measures long and have at least one stationary note.

5. *SUPPLEMENTA* MODELS

While examining stationary notes, it came to light that *supplementa* follow three different recurring models, and by necessity must consist of a minimum of three voices to sustain themselves as musical structures, attributes that were not mentioned by historical theorists previously. The first of these models consists of one stationary voice on a primary or secondary pitch, and we call it *S-1* — where *S* stands for *supplementum*, and the number represents an enumeration in order of frequency of occurrence. The *S-1* model is the most common model, and can be subdivided into 2 groups: (A) where a stationary pitch is the same as the *finalis* (primary pitch), and (B) where a stationary pitch is different from the *finalis*, i.e. secondary pitch, usually the *repercussa*. Models based on two or more stationary voices built on a primary pitch (same pitch class) we refer to *S-2*. The *supplementum* in the 'Salve nos, Domine' motet by Jean Mouton (Figure 4) is of the *S-2* type model. Models based on two or more stationary voices that may use a primary pitch, a secondary pitch, and sometimes a tertiary pitch (different pitch classes), the least common model, we refer to as the *S-3* type model.

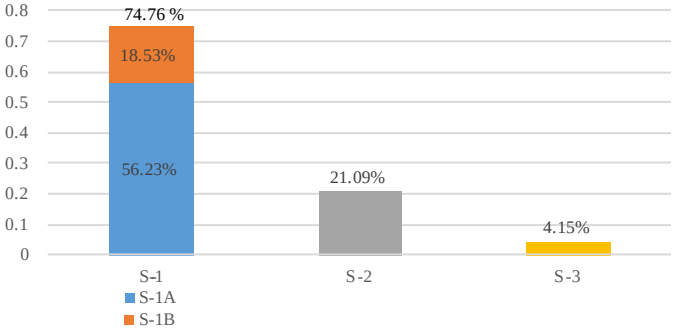

Fig. 7. Supplementa Model Distribution.

The chart in Figure 7 shows how the models are distributed within the *supplementa* dataset. Model *S-1* occurs 74,76 % of the time, where 75,21 % of model *S-1* — blue, or 56,23 % of total occurrences — are of type A having the same stationary pitch as the *finalis*. It is the more common type. The B type of model *S-1* (orange), where the single stationary pitch is not the finalis, occurs 24.79 % of the time — which makes about 18,53 % of total occurrences. Model *S-2* occurs 21,09 % of the time (gray). Here the stationary pitches (or pitch classes) are the same as the *finales* 100 % of the time. Model S-3 (yellow): occurs only 4,15 % of the time, the *finales* will be contained 100 % of the time within the stationary pitch group that consists of different pitch classes.

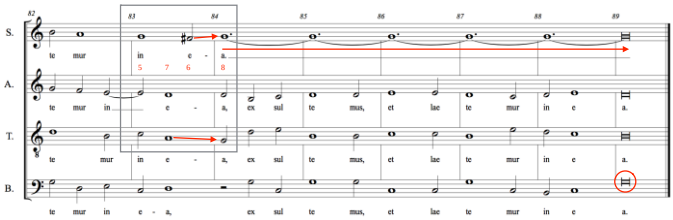

Fig. 8. Palestrina, 'Dies sanctificatus'.

In Palestrina's 'Dies sanctificatus' motet (Figure 8), a final cadence occurs from the second third of m. 83 to m. 84 with a 6–8 motion between the soprano and tenor voices. M. 84 is the beginning of the *supplementum*, as exemplified by the stationary G held in the soprano part. M. 89 shows that indeed G is the final. The example shows a *supplementum* of the *S-1 A* model variety — *supplementum* with a primary stationary pitch, and a *finalis* on G. The majority of all non-passing motions during the *supplementum* are indeed consonant as was described by Burmeister previously.

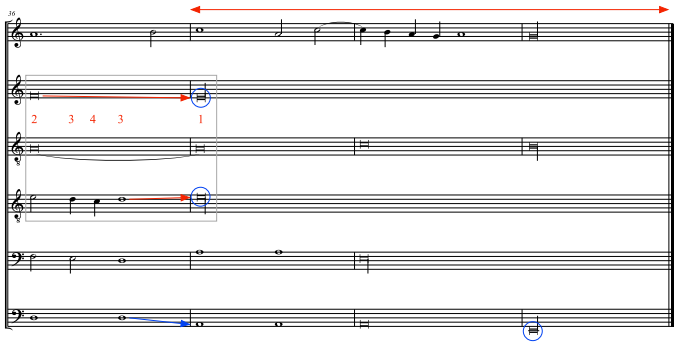

Fig. 9. Jean Mouton, 'Salve nos, Domine'.

A *S-2* type model *supplementum* appears in Moutons 'Salve nos, Domine' motet (Figure 9). The voices from top to bottom are: Superius, Altus, Tenor1, Tenor2, Bassus, and Bassus2. A final cadence occurs when a vertical motion from 3–1 is placed between the Altus and Tenor2 parts, supported by a half-step descending motion from F to E in the Altus part, and an ascending whole-step motion from a D to an E in the Tenor2 part. The elaboration of primary pitch E — in two voices here — becomes the stationary pitch. Because of the occurrence of two primary pitches the *supplementum* type is a *S-2* model. In addition, the sonority at the beginning of the *supplementum* is different from the sonority at the end of the *supplementum*. The *clausula in mi* strategy dictates this behavior, meaning the melodic bass motion from scale degrees 7 to 4 below the vertical third to unison succession, results in an A sonority, due to the Phrygian problem (parallel octaves). The *supplementum* allows the composer to put an E in the bass at the end of the piece.

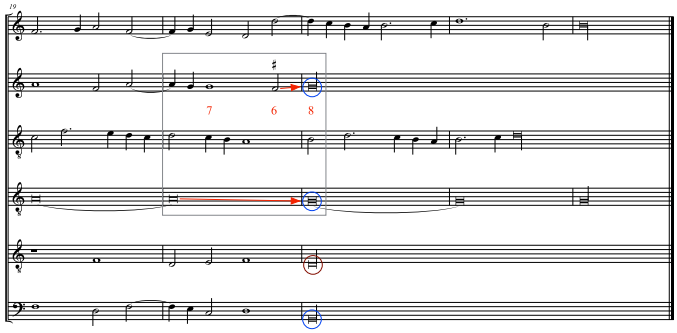

Fig. 10. Josquin?, 'In illo tempore stetit Jesus'.

A composition that represents the third *supplementum* model (*S-3*) is 'In illo tempore stetit Jesus', allegedly written by Josquin (Figure 10). The voices from the top down here are: Superius, Altus, Tenor1, Tenor2, Bassus1, Bassus2. A 6–8 motion occurs between the Altus and the Tenor2 lines. The 6–8 motion is supported by an ascending melodic half-step motion from F♯ to G in the Altus, and a descending melodic whole-step motion from A to G in the Tenor2. The goal note is G. The G is used as a stationary note in the Altus, Tenor2, but also the Bassus2 lines. However, the composer also adds a stationary D in the Bassus2 part, to enrich the timbre. The addition of this stationary note aside from the primary pitch makes the example a *S-3* model.

6. *FINALIS* RELATIONSHIPS

In addition to these three models, we found additional attributes about *supplementa* in regard to cadence pitch, the stationary note, and the lowest note in the final sonority (Figures 10). The cadence pitch is only 64,66 % of the times in agreement with the lowest note of the final sonority. The cadence pitch is about 74,30 % in agreement with its ensuing stationary note, while the stationary note is in agreement with the final sonority only about 63,05 % of the time. The next question that arises is whether or not this *phenomenon* is different for different types of modes — we will omit plagal modes for the time being —, especially in regard to pitch of cadence and pitch of the final bass notes.

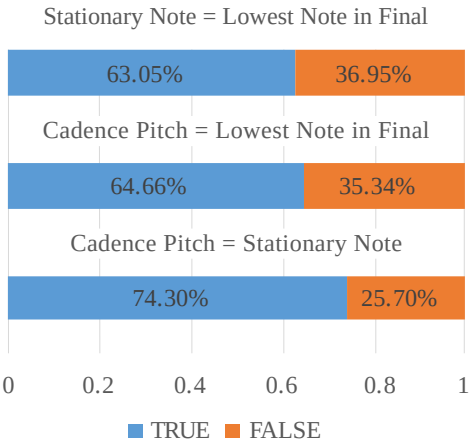

Fig. 11. *Finalis* Relationships in *Supplementa*.

In the following *supplementa* correlation matrices — Figure 12, representing motets with no key signature, and Figure 13, representing motets with one flat in the key signature —, we are tracking whether the pitch of the cadence (or PoC) is the pitch of the lowest note in the final vertical sonority (PLNFS) in more detail. The task is split among occurring key signatures. The diagonal grey boxes highlight where the pitch of cadence is equal to the pitch of the final vertical sonority in the bass.

No Key Signature

	D	E	F	G	A	C	PoC
D	10.74%	0.83%	0.00%	1.65%	1.65%	0.00%	14.88%
E	0.00%	22.31%	0.00%	0.00%	4.13%	0.83%	27.27%
F	0.00%	0.00%	2.48%	0.83%	0.00%	0.00%	3.31%
G	0.00%	0.83%	0.00%	15.70%	0.00%	0.83%	17.36%
A	7.44%	4.96%	0.00%	1.65%	14.05%	0.00%	28.10%
C	0.00%	0.83%	0.00%	0.83%	0.83%	6.61%	9.09%
PLNFS	18.18%	29.75%	2.48%	20.66%	20.66%	8.26%	100.00%

Pitch of Cadence (PoC) => Pitch of Lowest Note, Final Sonority (PLNFS)

Fig. 12. Correlation matrix of *Supplementa* with no signature.

Pieces without key signatures (Figures 12), have a broad spread. Almost a quarter of these *supplementa* in motets share E as the pitch of cadence (PoC) and the pitch of the lowest note in the final sonority. However, the rest of the motets are divided between G, A, D, and C, where the pitches of the cadences and the pitches of the lowest note of the final sonorities (PLNFS) are shared.

The most common final bass note in *supplementum* motets without a key signature, regardless of where the cadence originated from is E, followed by G, A, D, and C, mirroring our previous observation. However, the most common cadence

note, regardless of where it is going to is A, followed by E, G, D, C, and F in pieces without a key signature. We can observe that *supplementa* are not commonly used with F and C pieces — either with or without the cadence pitch sharing the pitch of the lowest note in the final sonority —, although C pieces certainly exist, supporting Glarean's argument for an Ionian mode. E cadences without a key signature have mostly E final bass notes, but a fairly good chunk of A final bass notes. G cadences without a key signature almost exclusively have G final bass notes. Pieces with A cadences — and no key signatures — most commonly have an A as the final bass note, followed by D and E — in order of frequency. D cadences without a key signature almost exclusively have D final bass notes.

These observances bring out the complexity of the relationships and modal classification for Aeolian and Phrygian pieces with no key signature. Perhaps a clear division and the existence of the Aeolian mode as opposed to the *tonus peregrinus* was not as pronounced as Glarean asserted.

1 Flat

	G	A	Bb	C	D	F	PoC
G	29.81%	0.96%	0.96%	2.88%	0.96%	0.00%	35.58%
A	0.00%	10.58%	0.00%	0.00%	1.92%	0.96%	13.46%
Bb	0.00%	0.00%	0.96%	0.96%	0.00%	0.96%	2.88%
C	0.00%	0.00%	0.00%	0.00%	0.00%	7.69%	7.69%
D	10.58%	1.92%	0.00%	0.00%	1.92%	0.00%	14.42%
F	0.00%	0.00%	0.00%	0.96%	0.00%	25.00%	25.96%
PLNFS	40.38%	13.46%	1.92%	4.81%	4.81%	34.62%	100.00%

Pitch of Cadence (PoC) => Pitch of Lowest Note, Final Sonority (PLNFS)

Fig. 13. Correlation matrix of *Supplementa* with one flat.

But what about motets with one flat in the key signature? Figure 13 shows a correlation matrix of pieces with one flat their key signatures. Almost a third of all *supplementa* motets of this type share a G with the cadence and the final bass note. A quarter of the pieces share a F with the cadence and the final bass note, and about 10 % of the pieces follow the same procedure with A. The G is the most common final bass note, or occurs 40 % of the time in pieces with one flat, regardless from where the *supplementum* originated, followed by F, A, C, and D. However, the most common cadence note, regardless of where it is going to is G, followed by F, D, A, and C. Pieces with one flat in the signature do usually not end on B♭ — either with or without the cadence pitch sharing the pitch of the lowest note in the final sonority. Therefore, D cadences with a flat signature almost exclusively will end with a G as the lowest pitch of the final sonority. Whereas C cadences almost exclusively will end with a F as the lowest pitch of the final sonority.

Supplementum motets with a flat signature are not necessarily simple transpositions, but have attributes unique to themselves. Also, *supplementum* motets with one flat key signature tend to avoid a B♭ cadence and final bass note endings altogether, but favor the G cadence as their final endings.

7. CONCLUSION

We have shown how the *supplementum* is defined by historical music theorists, and how the definitions of these theorists have been interpreted by modern scholars, and compared that information with actual data gathered by way of a corpus study. For the most part, historical theorists were not as far off with their contemplations. They were right about stationary pitches, and *supplementa* lengths. They made incomplete assumptions whether *supplementa* always occur at the end of a whole piece, and whether a *supplementum* confirms the pitch of the final cadence. In addition, historical theorists did not describe the three different *supplementa* models we have introduced here.

Within the different types of *supplementa* models, or types, we found that the pitch of the cadence is not necessarily the pitch of the final sonority, and that this varies according to the mode, especially in regard to pieces with no or one flat in their respective key signatures.

The combination of results will enable us to adjust our algorithms for different types of *supplementa*, and different types of cadences. Our corpus and its corresponding dataset can also function as ground truth for supervised machine learning, as we work toward a 'mode-finding algorithm' that works for real Renaissance music.

KEYWORDS

Renaissance, Motets, Supplementum, Counterpoint, Early Music, Polyphony, Structure, Analytical Theory, Music Analysis and New Technologies, Music Analysis and Music Theory, Quantitative Approaches to Analysis, Computational Musicology.

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